The Choice Is Yours

Comparing Polynomials in Different Representations

5.5

LEARNING GOALS

In this lesson, you will:

- Compare polynomials using different representations.
- Analyze key characteristics of polynomials.

Infinity refers to something that goes on forever. The set of natural numbers {1, 2, 3, . . .} and the set of integers {. . . −2, −1, 0, 1, 2 . . .} are examples of infinite sets because they continue without end. Another example of an infinite set is the set of rational numbers between 0 and 1.

Seeing different infinite number of sets begs the question: do all infinite sets have the same quantity of numbers in them? The set of natural numbers are only positive, while the set of integers are positive and negative. Does this mean that the set of natural numbers has fewer numbers than the set of integers?

How do you compare the size of these sets of numbers? Is it possible for one infinite set to be greater than another infinite set?

PROBLEM 1 The Best of Both Representations

Recall that you can represent a polynomial using a graph, table of values, equation, or description of its key characteristics. The ability to compare functions using different representations is an important mathematical habit. This skill allows you to model problems in different ways, solve problems using a variety of methods, and more easily identify patterns. At times you may need to compare functions when they are in different representations.

When comparing two functions in different forms, it may be helpful to ask yourself a series of questions. Examples include:

- *What information is given?*
- *What is the degree of each function?*
- *What do I know about all functions of this degree?*
- *What key characteristics do I need to know?*
- *How do the functions compare?*

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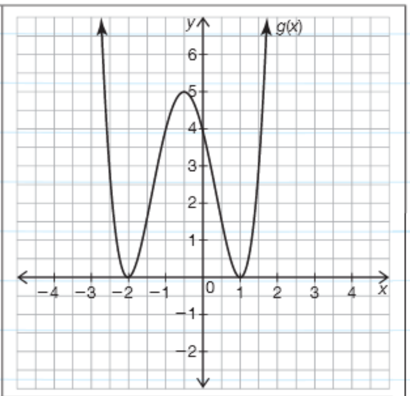
Consider two polynomial functions $f(x)$ and $g(x)$. Which polynomial has a greater number of real zeros? Justify your choice.

$f(x) = -2(x - 1)^3$

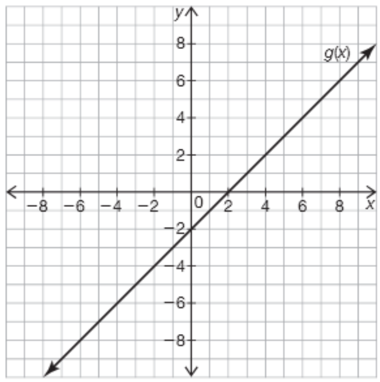


- The Fundamental Theorem of Algebra states that the number of zeros must be equal to the degree of the function. Therefore, $f(x)$ has 3 zeros.
- The function $f(x)$ has a real zero at 1 (multiplicity 3), so all zeros are real.
- The graph of $g(x)$ shows each zero has multiplicity 2, for a total of 4 real zeros.

The function $g(x)$ has 4 real zeros while $f(x)$ has 3. Therefore the correct choice is $g(x)$.

1. Toby compared the table of values for $f(x)$ and the graph of $g(x)$ to determine which polynomial function has the greater number of real zeros.

x	$f(x)$
−2	3
−1	−2
0	−5
1	−6
2	−5
3	−2
4	3



Toby

Function $g(x)$ has the greater number of real zeros. The graph has 1 zero at $x = 2$ while the table of values has no output value of 0, and therefore no zeros.

Is Toby correct? Explain your reasoning.

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2. Analyze each pair of representations. Then, answer each question and justify your reasoning.

a. Which function has a greater degree?

A polynomial function $h(x)$ has 1 absolute maximum and 1 relative maximum.	$j(x) = -40(x - 7)^2 + 30x^2 - 17x + 1$

b. Which function has a greater degree?

x	$m(x)$
−2	9
−1	3
0	1
1	3
2	9

A polynomial function $n(x)$ has a real zero and an imaginary zero.

c. Which function has a degree divisible by 2?

x	$p(x)$
−2	2
−1	4
0	6
1	8
2	10

The function $q(x)$ has only imaginary solutions.

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d. Determine which function has the greater output as x approaches infinity.

An odd function $r(x)$ with $a < 0$.	$k(x) = x^6 + x^4 + 3x^2 + 5x - 10{,}000$

e. Determine which function has the greater output as x approaches negative infinity.

$t(x) = -3(x - 4)^8 + 130$	A quartic function $s(x)$ with y-intercept (0, 5) and all imaginary roots.

3. Sam and Otis disagree when they compared the two functions shown to determine which one has an odd degree.

The function $f(x)$ has an absolute maximum value.	$g(x) = x^4(3 - x)(2x^2 + 3)(x^4 + 4)$

Sam
The function f(x) has an odd degree because odd functions approach positive infinity as x either increases or decreases. This means f(x) has a maximum value.

Otis
The function g(x) has an odd degree. When I multiplied the factors, I got a term with a highest exponent of 11:
$x^4(-x)(2x^2)(x^4) = -2x^{11}$.
Therefore, g(x) is odd.

Who is correct? Justify your reasoning.

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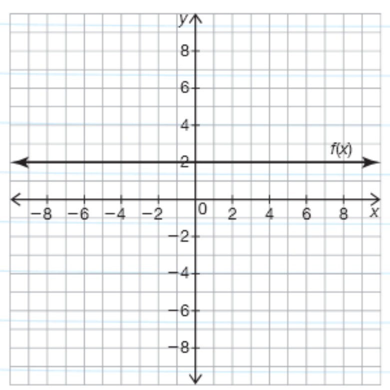
PROBLEM 2 Two Representations Are Better Than One

Many problems in mathematics are unique, without specific step-by-step algorithms that lead to an answer. In Problem 1, *The Best of Both Representations*, you mentally asked yourself a series of metacognitive questions to compare functions in different representations. As you consider additional questions in this lesson, it may be helpful to compare the problems to ones that you have already completed.

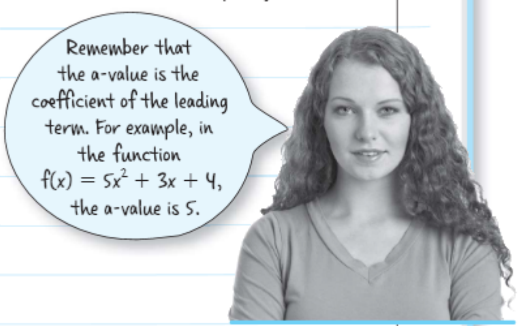
Ask yourself:

- *How is this problem the same or different than the previous ones that I have already solved?*
- *What do I know about the function that is given? What can I conclude that is not directly stated?*

Consider the representations shown. Which function has a greater y-intercept? Justify your reasoning.



A function $g(x)$ has an a-value less than zero and all roots have a multiplicity of 2.



Solution:

This problem is similar to previous problems in that you must consider functions with restrictions on the a-value and functions with multiple roots. The problem is also similar in that you must consider an output value for a given input. In this case, the input is 0.

In function $f(x)$, the output value is 2 for any given input. Analyzing function $g(x)$, the multiplicity 2 tells you that the function is even, and the negative a-value indicates that the function opens downward. The multiplicity of the roots also tells you that the function does not cross the x-axis. Instead, it reflects at a given point where the double root occurs.

Comparing the two functions, you know that function $g(x)$ is always below the x-axis and function $f(x)$ is above the x-axis. Therefore, $f(x)$ has a greater y-intercept.

1. Isaac and Tina disagree over which function has a greater y-intercept.

$g(x) = 2(x - 2)(x + 2)(x - 3) - 4$

x	$h(x)$
−2	−2
−1	0
0	4
1	10
2	18

Isaac
Function g(x) has a greater y-intercept. I calculated the y-intercept by substituting 0 for x. This value is greater than (0, 4) shown in the table for the function h(x).

Tina
Function h(x) has a greater y-intercept. The y-intercept of h(x) is (0, 4) and the y-intercept of g(x) is (0, −4).

Who is correct? Justify your reasoning.

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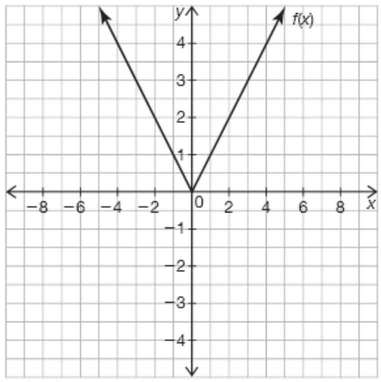
2. Analyze each pair of representations. Then, answer each question and justify your reasoning.

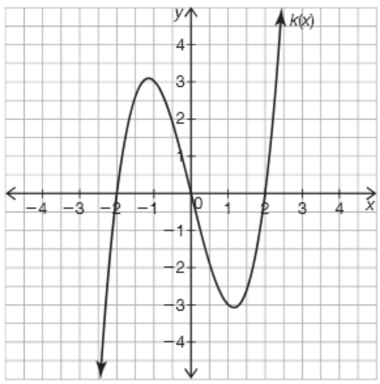
a. Which function has a greater average rate of change for the interval (−4, 4)?



$g(x) = x$

b. Which function has a greater average rate of change for the interval (−1, 1)?

x	$j(x)$
−2	4
−1	1
0	0
1	1
2	4



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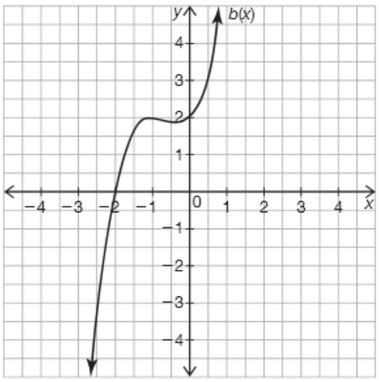
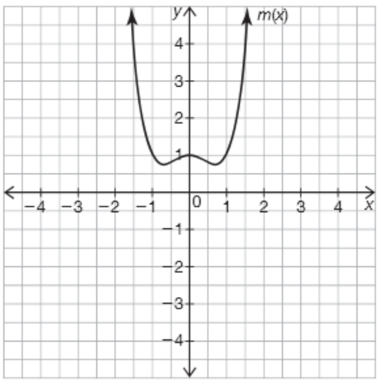
c. Which function has a greater relative minimum?

A cubic function $a(x)$ with $a > 0$ and 3 distinct real roots.	

d. Which function's axis of symmetry has a greater x-value?

	$n(x) = x^2 - 3x + 1$

3. Emilio studied the table of values and description of the key characteristics to determine which function has a greater minimum.

x	$d(x)$
−2	5
−1	2
0	1
1	2
2	5

A quartic function $m(x)$ has $a < 0$ and 2 pairs of real zeros (multiplicity 2).

Emilio

Function $d(x)$ has a greater minimum. This function is a parabola opening up, with its vertex at (0, 1). Function $m(x)$ opens down because $a < 0$. Since the real zeros have multiplicity 2, I know any real zeros occur when the function reflects off of the x-axis. Therefore, the output values of $m(x)$ never reach a point greater than $y = 0$.

Is Emilio correct? Justify your reasoning.

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Recall that a basic function is a function in its simplest form. The basic function of *a* is $f(x) = x^n$ for any natural number n. Transformations of the basic functions are performed by changing the A-, B-, C-, and D-values in the form $g(x) = Af(B(x) - C) + D$. Remember, each value describes different transformations of the graph: the A-value vertically stretches or compresses the graph, the B-value horizontally stretches or compresses the graph, the C-value horizontally shifts the graph right or left, and the D-value vertically shifts the graph up or down.

4. Analyze the transformations of the basic functions. Then answer each question and justify your reasoning.

a. Which function has a greater output for a given input?

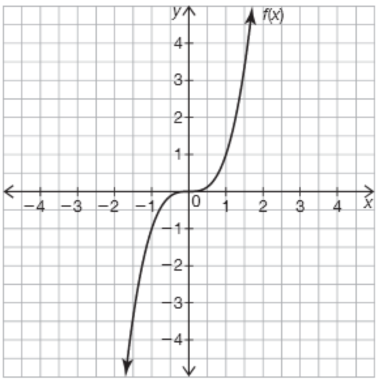
The basic quadratic function $f(x) = x^2$.	$g(x) = f(x - 2) + 1$

b. Which function has a lower minimum?

x	$j(x)$
−2	16
−1	1
0	0
1	1
2	16

$k(x) = 5f(x - 4) + 2$

c. Which function has the greater input for a given output value?



$g(x) = 3f(x - 5) + 1$

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Be prepared to share your solutions and methods.